Analysis and Comparison of Speed Control of DC Motor using Sliding Mode Control and Linear Quadratic Regulator

[1]Satyabrata Sahoo | [2]Gayadhar Panda
[1](Asst. Professor, Department of Electrical Engineering, C.V. Raman College of Engineering, Bhubaneswar, Odisha. email: jitu_sahoo@yahoo.com)
[2](Asso. Professor, Department of Electrical Engineering, National Institute of Technology Meghalaya. email: p_gayadhar@yahoo.com)

Abstract—*The objective of this paper is to control the angular speed* ω_m *in a model of a DC motor using different control strategies for comparison purpose. The comparison is made between different controllers. The controllers are closed loop unity feedback controller, PID controller and the state feedback controllers like linear quadratic regulator design based on the optimal control theory and sliding mode control. Performance of these controllers has been verified through simulation results using MATLAB/SIMULINK software.*

Keywords—model-based control; PID Controller; LQR control method; sliding mode control (SMC); state-space models, DC motor; optimal control

1. INTRODUCTION

Control system design refers to the process of selecting feedback gains that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant.

Due to the excellent speed control characteristics of a DC motor, it has been widely used in industry (such as cars, trucks and aircraft) even though its maintenance costs are higher than the induction motor. As a result, authors have paid attention to position control of DC motor and prepared several methods to control speed of such motors. Proportional–Integral-Derivative (PID) controllers have been widely used for speed and position control [1].

They designed a position controller of a DC motor by selection of PID parameters using genetic algorithm and secondly by using Ziegler & Nichols method of tuning the parameters of PID controller. They found that, first method gives better results than the second one.

In [2] they have compared two types of controllers which are PID controller and optimal controller. The PID compensator is designed using (GA), while the other compensator is made optimal and integral state feedback controller with Kalman filter. Computer simulations have been carried out. Finally they found that the second controller gives less settling, less overshoot and better performance encountering with noise and disturbance parameters variations.[3] presented a novel PID dual loop controller for a solar photovoltaic (PV) powered industrial permanent magnet DC (PMDC) motor drive. MATLAB/SIMULINK was used in the analysis for the GUI environment.

The performance of PI controller for speed or position regulation degrades under external disturbances and machine parameter variations. Furthermore, the PI controller gains have to be carefully selected in order to obtain a desired response. This can be solved by advanced control techniques such as sliding mode control.

This paper presents Sliding Mode Control, LQR and PID controller which applied to control the speed of a DC motor. The rest of the paper is presented as follows: at first the plant model is described. The next section describes the PID technique, the design of LQR and Sliding Mode Control. Then simulation results are presented. Finally, the last section contains paper conclusion.

2. DC MOTOR MODEL

The speed of a DC motor is proportional to the voltage applied to it. While, its torque is proportional to the motor current. Speed control can be achieved by variable battery tapings, variable supply voltage, resistors or electronic controls.

A simple motor model is shown in Fig.1. The armature circuit consist of a resistance (Ra) connected in series with an inductance (La), and a voltage source (eb) representing the back emf (back electromotive force) induced in the armature when during rotation. [4].

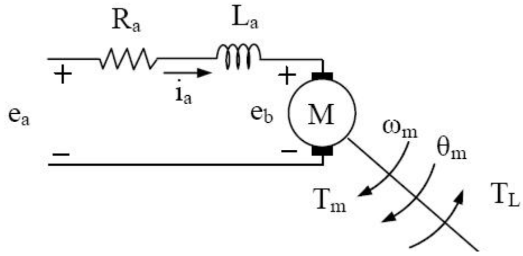

Fig. 1 DC Motor Model [4]

The motor torque T_m is related to the armature current, i_a , by a torque constant K_i;

$$T_m = K_i i_a \quad (1)$$

The back emf, e_b, is relative to angular velocity by;

$$e_b = k_b \omega_m = k_b \frac{d\theta}{dt} \quad (2)$$

From Fig. 1 we can write the following equations based on the Newton's law combined with the Kirchoff's law:

$$e_a = i_a R_a + L_a \frac{di_a}{dt} + e_a \quad (3)$$

$$L_a \frac{di_a}{dt} + R_a i_a = e_a - K_b \frac{d\theta}{dt} \quad (4)$$

$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = K_i i_a \quad (5)$$

There are several different ways to describe a system of linear differential equations. The plant model will be introduced in the form of state-space representation and given by the equations:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \quad (6)$$

According to equations from (2) to (5), the state space model will be:

$$\begin{bmatrix} \dot{i_a} \\ \dot{\omega_m} \\ \dot{\theta_m} \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -K_b/L_a & 0 \\ K_i/J_m & -B_m/J_m & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix} e_a \quad (7)$$

$$\omega_m = [0 \quad 1 \quad 0] \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} \quad (8)$$

The DC motor data taken for this work are [5]:

Symbol	Value and unit
E	= 12volt
J_m	= 0.01kgm^2
B_m	= 0.00003kgm^2/s
K_i	= 0.023Nm/A
K_b	= 0.023V/rad/s
R_a	= 1Ω
L_a	= 0.5H

3. LQR & STATE FEEDBACK OBSERVER CONTROLLER

LQR control that designed is classified as optimal control systems. This is an important function of control engineering.

The performance of a control system can be represented by integral performance measures. Therefore, the design of the system must be based on minimizing a performance index, such as the integral of the squared error (ISE).

The specific form of the performance index can be given as in eq.(9), where x^T indicates the transpose of the x matrix, then, in terms of the state vector, is

$$J = \int_0^{t_f} (x^T x) dt \quad (9)$$

Where x equals the state vector, and t_f equals the final time. Then the design steps are as follows:

1. Determine the matrix P that satisfies eq.(10), where A is known.

$$A^T P + PA = -I \quad (10)$$

2. Minimize J by determining the minimum of eq.(11) by adjusting one or more unspecified system parameters.

$$J = \int_0^{\infty} x^T x \, dt = x^T(0) P x(0) \quad (11)$$

Upon examining the performance index (eq.11), recognizing that the reason the magnitude of the control signal is not accounted for in the original calculation is that U (equals the control vector) is not included within the expression for the performance index.

To account for the expenditure of the energy of the control signal, it will be utilize the performance index

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (12)$$

Where Q is a positive definite or positive semi definite Hermitian matrix.

and R is a positive definite Hermitian matrix. Q & R are weighing factors. The performance index J can be minimized when

$$K = R^{-1} B^T P \quad (13)$$

From Fig.3 the state variable feedback will be represented by

$$u = -Kx \quad (14)$$

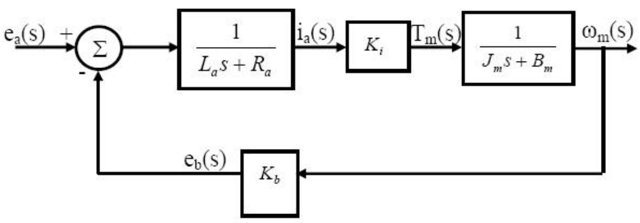

Fig. 2- DC-Motor System Block Diagram for speed

The n×n matrix P is determined from the solution of equation

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \qquad (15)$$

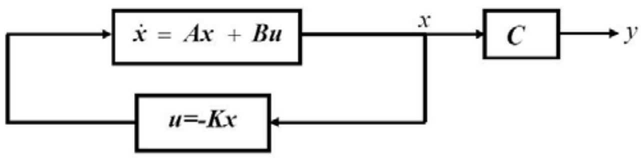

Fig. 3- Linear Quadratic Structure

The two matrices Q and R are selected by design engineer by tray and error. Generally speaking, selecting Q large means that, to keep J small. On the other hand selecting R large means that the control input u must be smaller to keep J small.

Equation (15) can be easily programmed for a computer, or solved using MATLAB. Popularly equation (15) is known as Algebraic Riccati Equation.

4. SLIDING MODE CONTROL

Sliding mode control is a nonlinear control strategy which is a special form of Variable structure system. Unlike other nonlinear control like state feedback law etc it's not a continuous control law but a discontinuous one [6].

A control system can be described as:

$$\dot{x} = f(x,u,t)\ x\epsilon R^n, u\epsilon R^n, t\epsilon R \qquad (16)$$

The switching function is given by v = cx1 + x2 and the line v = 0 is the surface on which the control u has the discontinuity as seen in Fig. 4. The discontinuous control may be considered as:

$$u = \begin{cases} u^+, v > 0 \\ u^-, v < 0 \end{cases} \qquad (17)$$

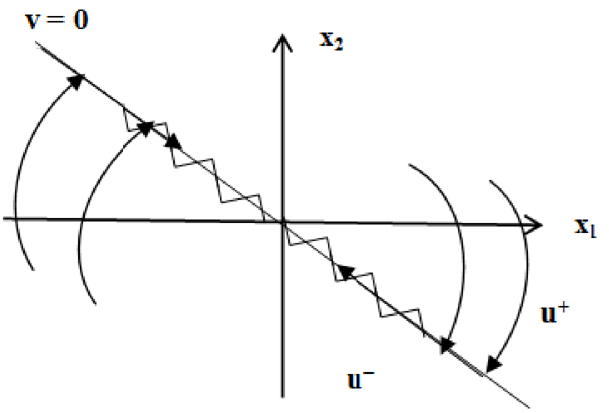

Fig. 4 Sliding Mode Control

It is seen clearly that the state reaches the switching line in finite time. The state crosses the v=0 line resulting in the value of u being altered from u+ and u- . The system parameter and C will decide whether the trajectory will continue in the other side of v< 0 or not. There can be the other situations where trajectory will recross the switching line for sliding motion to occur when following conditions are met.

$$\lim_{v \to 0^+} \dot{v} < 0\ and\ \lim_{v \to 0^+} \dot{v} > 0 \qquad (18)$$

Let a Single input nonlinear system be defined as $x(n) = f(x,t) + b(x,t)u(t)$, where x(t) is the state vector , u(t) is the control input and n is the order of differentiation. Though $f(x,t)$ and $b(x,t)$ are nonlinear in general but are bounded in the sense that their bounds are known [7]. A surface which varies with time is defined in state space and equated to zero, is given by

$$S(x,t) = \left(\frac{d}{dt} + \delta\right)^{n-1} \tilde{x}(t) = 0 \qquad (19)$$

Here δ being positive considered as Bandwidth (BW) of the system and $\tilde{x}(t) = x(t) + x_d(t)$ is the error.

5. SLIDING SURFACE DESIGN

The state space model of a separately excited DC motor is obtained as shown in Equation (7). The speed control goal is to force the speed ω_m to track the desired speed referenceω_d. For the sliding mode controller technique, the sliding surface is chosen as:

$$s = \dot{\omega_e} + \delta\omega_e \qquad (20)$$

Where ω_e is the tracking speed error. δ is a strictly positive constant that determine the bandwidth of the system. The given speed control problem can be treated as a regulator problem, where the desired acceleration is chosen to be zero.

6. SIMULATION RESULTS

The simulation procedure may be summarized as follows:

- First input the DC motor data
- Write the differential equations for the model then get the state space representation as in equation (7).
- Get the open loop transfer function and the closed loop step response.

Finally performing the performance of PID controller, LQR controller, Observer based controller [8], sliding mode control and then compare the results.

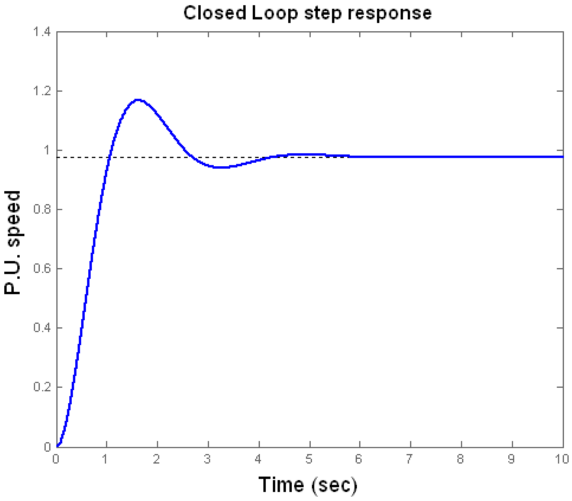

Fig. 5-Closed loop step response

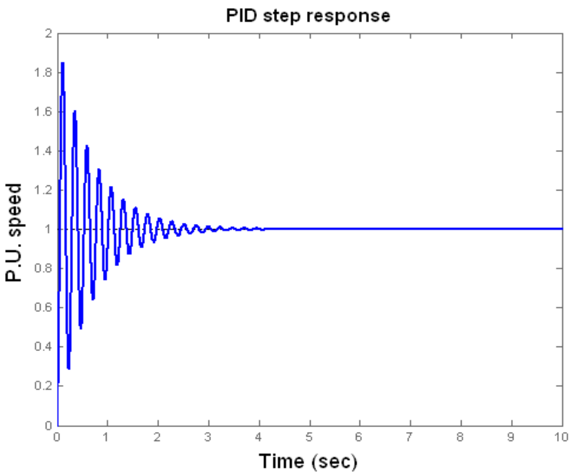

Fig. 6-PID step response

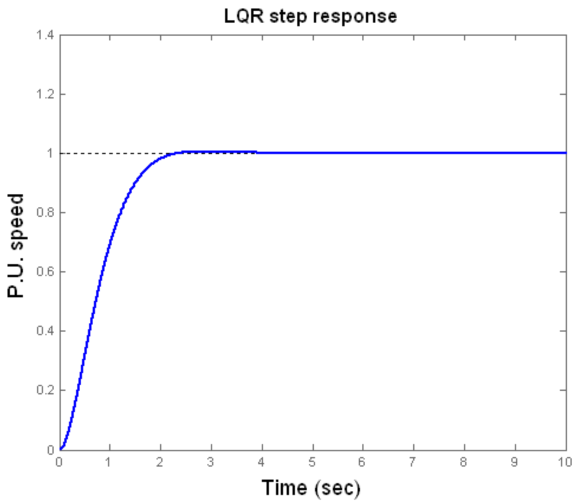

Fig. 7-LQR step response

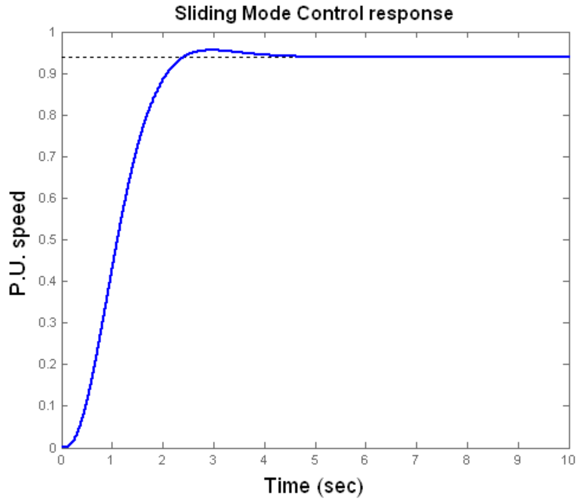

Fig. 8- Sliding Mode Control Response

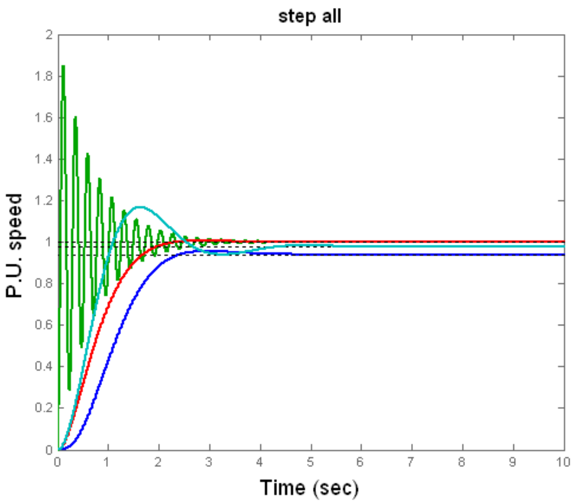

Fig. 9- step response with all controllers

Simulation results for the SMC is presented in Figure 8, which shows the rotor speed responses for SMC controller when a separately excited DC motor is operating at a reference speed of 10 rad/s. The sliding mode controller shows a little overshoot which is reasonable and then tracks the reference speed closely.

7. CONCLUSION

Speed control of a DC motor is an important issue, so this paper presents a design method to determine the optimal speed control using different controllers. From the obtained results, which are shown in Table 1, we find LQR controller has smaller overshoot and sliding mode controller has shorter settling time than that of the other controllers other than LQR.

TABLE-1 COMPARISON OF SIMULATION RESULTS

Different Controllers	Settling Time In sec	Peak Amplitude	Over Shoot In %
Closed Loop With unity Feedback	3.83	1.17	19.5
PID Controller	2.76	1.85	84.8
LQR Controller	1.99	1.00	0.525
Sliding Mode Control	2.22	0.955	1.7

LIST OF SYMBOLS:

A = n×n constant matrix
B = n×1 constant matrix
B_m = viscous friction coefficient (kgm^2/s)
C = 1×n constant matrix
D = Constant
$e_a(t)$ = applied voltage (V)
$e_b(t)$ = back emf (V)
$i_a(t)$ = armature current(A)
J_m = moment of inertia of rotor ($kg.m^2$)
K_b = back emf constant (V/rad/s)
K_i = torque constant (Nm/A)
L_a = armature inductance (H)
R_a = armature resistance (Ω)
t_f = final time(sec)
$T_L(t)$ = load torque (Nm)
$T_m(t)$= motor torque (Nm)
u = control signal
χ = state vector
y = output signal
$\theta_m(t)$= rotor displacement (rad)
$\omega_m(t)$= rotor angular velocity (rad/s)
δ = Bandwidth of system
v = sliding surface

REFERENCES

[1] H. Neenu Thomas and Dr. P.Poongodi, "Position Control of DC Motor Using Genetic Algorithm Based PID Controller", Proceedings of the World Congress on Engineering 2009 Vol. II, WCE2009, July 1-3, 2009, London, U.K.

[2] Hadi Delavari, Ghasem Alizadeh and Mohammad Bagher Bannane Sharifian, "Optimal Integral State Feedback Controller for a DC Motor", 2006, paper identification number 440.

[3] Sharaf, A.M., Elbakush, E., Altas, I.H., "Novel Control Strategies for Photovoltaic Powered PMDC Motor Drives', IEEE, 2007

[4] Katsuhiko Ogata: Modern Control Engineering; Prentice Hall International, Inc. Fourth Edition 2002.

[5] Tutorial 12 about DC motor, instrumentation and process control by Hung Nguyen, 2006.

[6] Sliding mode control:Theory and applications, C. Edwards, s spurgeon

[7] xing-gang, y.; spurgeon, s. k.; edwards, E. Sliding mode control for time-varying delayed systems based on a reduced-order observer. Automatica. August 2010, vol. 46, no. 8, pp. 1354-1362. ISSN 0005-1098.

[8] Ashish Tiwari: Modern Control Design With Matlab and Simulink; John WILEY & Sons, Ltd., Singapore.