# MAGNETO HYDRODYNAMIC CONVECTIVE HEAT AND MASS TRANSFER IN A MICRO-POLAR FLUID

M Vinoop Reddy[1] | N Viswa Sairam[2]
[1](Kommuri Pratap Reddy Institute of Technology, Asst.Prof, vinoopreddy@gmail.com)
[2](MIC College of Technology, Asst Prof, nnviswasairam@gmail.com)

***Abstract***— *In general, analysis is made on any fluid considering it as a Newtonian fluid. Therefore there is a large variation between theoretical and practical results. In this paper, behavior of a micro-polar fluid which is a non-Newtonian fluid is presented by introducing different terms in navier-stokes and energy equations which exactly govern the flow of fluid. The micro-polar fluid considered is used as a coolant in the plate-type heat exchangers where hot fluid flows on one side and micro-polar fluid flows on the other side. Magnetic field is applied perpendicular to the direction of motion of fluid which contributes to the rotation of fluid particles there by reducing the shear stress. Hence effective heat transfer takes place in the hydrodynamic boundary layer requiring low pumping power and mass flow rate.*

***Keywords***— ***Newtonian fluid, Micro-polar fluid, Shear stress, Magnetic field***

## 1. Introduction

In fluid mechanics and heat transfer, ideal fluids are considered which obey Newton's law of viscosity. Shear stress is directly proportional to strain in case of Newtonian fluids. But the behavior of real fluids cannot be completely described by this law. Experiments on fluids that contain extremely small amount of polymeric additives indicated that the skin-friction near the rigid surface is about 30% to 50% lower than those without additives which are clearly described by Eringen[1]and Lukaszewicz G[2]. Such discrepancies in experimental results and theoretical predictions have been attributed several factors, including surface effects and micro-rotational effects of molecules. These experimental findings seem to imply that Navier-Stokes equations can no longer predict fluid behavior accurately in the above mentioned situations. Free convection heat and mass transfer past a moving vertical porous plate are indicated by Alam MS, Rahman MM, Sattar MA[3].Also the heat and mass transfer in MHD free convection from a moving permeable vertical surface by a perturbation technique is clearly mentioned in[4].The inadequacy of the classical continuum theory has led to the development of theories of micro-continua in which continuous media possess not only mass and velocity but also substructure. The behaviour of incompressible micro-polar fluid is described in [5]. Also, there exist different approaches to study the mechanics of fluids with a substructure. Several theories such as the theory of anisotropic fluids, the theory of simple micro fluids and polar fluids have been developed to take into account the geometry, deformation and intrinsic motion of individual material particles.

## 2. Micro-Polar Fluid

The theory of simple micro fluids stems from the concept of a micro continuum, which is a continuous collection of deformable point particles. In a micro continuum, material particles (or micro element) are said to be contained within a particle (or macro elements). As a result macro element is deformable. In a simple micro fluid, its properties and behaviour are also affected by local motion of the material particles (or microelements) contained in each of its particle (or macro element).The heat transfer taking place in micro-polar fluid is mentioned by Takhar HS[6].The heat transfer within the micropolar fluid is included in [7] and the convection process taking place between surface and micropolar fluid are indicated by Gorla RSR, Slaouti A[8]. The theory of simple micro fluids is a complicated one. Even in the simplest case of constitutively linear theory, a micro fluid has twenty-two viscosity coefficients. The theory of micro-polar fluids asserts that micro-polar fluids can support couple stresses and body couples and exhibit micro rotational effects. Micro-polar fluids can also model anisotropic fluids, liquid crystals with rigid molecules, magnetic fluids, cloud with dust, muddy fluids and other biological fluids. The theory of micro-polar fluid requires a transport equation representing the principle of conservation of local angular momentum to the usual transport equations for the conservation of mass and momentum with additional local constitutive parameters . This theory allows for two independent vectors, velocity vector $\bar{q}$ and microrotation vector $\bar{\omega}$ associated with each fluid particle. The microrotation vector represents the rotation in an average sense of the rigid particles centered in a small volume element about the centroid of the element. The field equations of micro-polar fluid dynamics are

$$\frac{\partial\rho}{\partial t} + \rho\nabla.\bar{q} = 0 \qquad (1)$$

$$\rho\frac{\partial\bar{q}}{\partial t} = \rho\bar{f} - \nabla p + \kappa\nabla X\bar{\omega} - (\mu+\kappa)\nabla X\nabla X\bar{q} + (\lambda + 2\mu + \kappa)\nabla(\nabla.\bar{q}) \qquad (2)$$

$$\rho j \frac{\partial \overline{\omega}}{\partial t} = \rho \overline{l} - 2\kappa\overline{\omega} + \kappa\nabla X \overline{q} - \gamma\nabla X\nabla X\overline{\omega} + (\alpha + \beta + \gamma)\nabla(\nabla.\overline{\omega}) \quad (3)$$

## 3. MAGNETO HYDRODYNAMICS

The interaction of electromagnetic fields and fluids can be described scientifically by the proper application of the principles of the special theory of relativity. The study of these applications to continuum is known as Magneto-Hydro-Dynamics (MHD). The study of MHD plays an important role in agriculture, engineering and petroleum industries, for instance, it may be used in the equipments such as nuclear reactors by liquid sodium and induction flow water which depends on the potential difference of the fluid in the direction perpendicular to the motion and goes to the magnetic field. The study of magneto-hydrodynamics of viscous conducting fluids is playing a significant role, owing to its practical interest and abundant applications. The concept of magneto-hydro-dynamics of viscous conducting fluids is playing a significant role, owing to its innumerable applications.

## 4. MATHEMATICAL FORMULATION

Consider a fluid flow which is steady, laminar, incompressible and two-dimensional. In the fluid, let free convective heat and mass transfer is taking place along a semi-infinite vertical plate embedded in a doubly stratified, electrically conducting micro-polar fluid. Choose the coordinate system in such a way that x-axis is along the vertical plate and y-axis is normal to the surface of plate. The physical model and coordinate system are shown in Fig. 1. The temperature variation along the length of plate is given by T(x) and concentration by C(x). The variation of temperature and the mass concentration of the surrounding medium are assumed to be linearly stratified in the form T∞ (x) =T∞, 0 + A (x) and C∞(x) =C∞, 0 + B(x), respectively, where A and B are constants and T∞,0 and C∞,0 are the beginning ambient temperature and concentration at x = 0, respectively. Consider a uniform magnetic field B0 applied in the direction perpendicular to the plate. The induced magnetic field is so high that the Reynolds magnetic number can be neglected.

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} = 0 \quad (4)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \left(\frac{\kappa}{\rho}\right)\frac{\partial \bar{\omega}}{\partial \bar{y}} + g^*[\beta_T(\bar{T} - \bar{T}_\infty)] + \beta_c(\bar{C} - \bar{C}_\infty) - \sigma\frac{B_0^2}{\rho}\bar{u} \quad (5)$$

$$\bar{u}\frac{\partial \bar{w}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} = \left(\frac{\gamma}{\rho j}\right)\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{\kappa}{\rho j}\left(2\omega + \frac{\partial u}{\partial y}\right) \quad (6)$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (7)$$

$$\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \quad (8)$$

## 5. METHOD OF SOLUTION

Introducing the following non-dimensional terms to non-dimensionalize the governing equations. These terms are substituted into the above equations

$$x = \frac{\bar{x}}{L};\ y = \frac{\bar{y}}{L};\ u = \frac{\bar{u}L}{\upsilon Gr^{1/2}};\ v = \frac{\bar{v}L}{\upsilon Gr^{1/4}};\ w = \frac{\bar{w}L}{\upsilon Gr^{3/4}}$$

$$Gr = \frac{g^*[\beta_T \delta T L^3]}{\upsilon^2};\ \theta = \frac{\bar{T} - T_\infty}{T_w(x) - T_\infty};\ \emptyset = \frac{\bar{C} - C_\infty}{C_w(x) - C_\infty}$$

We obtain the following equations which are dimensionless.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (1+K)\frac{\partial^2 u}{\partial y^2} + \kappa\frac{\partial \bar{\omega}}{\partial \bar{y}} + x(\theta + \mathfrak{B}\phi) - Mu \quad (9)$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \left(1 + \frac{K}{2}\right)\frac{\partial^2 \omega}{\partial y^2} + \kappa\left(2\omega + \frac{\partial u}{\partial y}\right) \quad (10)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + \frac{u\theta}{x} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + \frac{u\phi}{x} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \quad (12)$$

The boundary conditions are:

$$\bar{u} = 0,\ \bar{v} = -V_0,\ \bar{\omega} = 0,\ \bar{T} = T_w(\bar{x}),\ \bar{C} = C_w(\bar{x}) \ \text{at}\ \bar{y} = 0$$
$$\bar{u} \to 0,\ \bar{\omega} \to 0,\ \bar{T} \to T_\infty,\ \bar{C} \to C_\infty \ \text{as}\ \bar{y} \to 0$$

Where $K = \frac{\kappa}{\mu}$ is the Coupling number, $\mathfrak{B} = \frac{\beta_c \Delta C}{\beta_T \Delta T}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2 L^3}{\rho \upsilon^2 Gr}$ is the magnetic parameter, $Pr = \frac{\vartheta}{\alpha}$ is the Prandtl number and $Sc = \frac{\upsilon}{D}$ is the Schmidt number.

Introducing the following Lie Group transformations,

$$x = \hat{x}e^{-\varepsilon\alpha_1};\ y = \hat{y}e^{-\varepsilon\alpha_2};\ \psi = \hat{\psi}e^{-\varepsilon\alpha_3};\ \omega = \hat{\omega}e^{-\varepsilon\alpha_4};\ \theta = \hat{\theta}e^{-\varepsilon\alpha_5};\ \phi = \hat{\phi}e^{-\varepsilon\alpha_6}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are transformation parameters and $\varepsilon$ is a small parameter, this scaling group of transformations transform coordinates $(x, y, \psi, \omega, \theta, \phi)$ to $(\hat{x}, \hat{y}, \hat{\psi}, \hat{\omega}, \hat{\theta}, \hat{\phi})$ and above five equations and their boundary conditions are invariant under the point transformations , along with the stream function $\psi$ such that

$$u = \frac{\partial \psi}{\partial y};\ v = -\frac{\partial \psi}{\partial x}$$

Hence, the transformed governing equations are

$$(f')^2 - ff'' = (1+K)f''' + Kg' + \theta + \mathfrak{B}\phi - Mf' \quad (13)$$

Skin Friction And Wall Couple Stress

The aim of this study is to estimate the skin friction coefficient C_f, local heat transfer coefficient, Nusselt number 〖Nu〗_x.The non dimensional skin friction coefficient, local heat-transfer coefficient and mass transfer coefficient are given by

$$\tau_w = \left[(\mu + k)\frac{\partial u}{\partial y} + k\omega\right]_{y=0} \quad ; \quad m_w = \gamma\left[\frac{\partial \omega}{\partial y}\right]_{y=0} \qquad (15)$$

The non-dimensional skin friction $C_f = \frac{2\tau_w}{\rho A^2}$ and wall couple-stress $M_w = \frac{Bm_w}{\rho A^2}$, where A is the characteristic velocity, are given by

$$C_f = \left(\frac{2}{1-N}\right) f''(0)\bar{x} \text{ and } M_w = \left(\frac{2}{1-N}\right)\frac{\lambda}{J} g'(0)\bar{x} \qquad (16)$$

$$\text{where } \bar{x} = Bx$$

The heat and mass transfers from the plate, respectively, are given by

$$q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0} \text{ and } q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

## 6. HEAT AND MASS TRANSFER RATES

The non-dimensional rate of heat transfer, called the Nusselt number $Nu = \frac{q_w}{Bk(T_w - T_\infty)}$ and rate of mass transfer,called the Sherwood number $Sh = \frac{q_m}{BD(C_w - T_\infty)}$ are given by

$Nu = -\theta'(0)$ and $Sh = -\phi'(0)$

## 7. RESULTS AND DISCUSSION

The non-linear non-homogeneous ordinary differential equations are solved numerically using the Keller-box implicit method. This method has a second order accuracy and unconditionally stable. The calculations are repeated until a pre-defined convergent value is obtained and the calculations are stopped when

$\delta f''(0) \le 10^{-8}$, $\delta\theta'(0) \le 10^{-8}$ and $\delta\phi'(0) \le 10^{-8}$. In order to see the effects of step size ($\Delta\eta$ ) we executed the code for our model with three different step sizes as $\Delta\eta$ = 0.001, $\Delta\eta$ = 0.01 and $\Delta\eta$ = 0.05 and in each case we

$$f'g - fg' = \left(1 + \frac{k}{2}\right)g'' - 2Kg - Kf'' \qquad (14)$$

$$\frac{1}{P_r}\theta'' - \theta f' + f\theta' - \varepsilon_1 f' = 0$$

$$\frac{1}{S_c}\phi'' - \phi f' + f\phi' - \varepsilon_2 f' = 0 \qquad (17)$$

Found very good agreement between the results. After some trials we imposed a maximal value of h (i.e., $\eta$ value) as 6 and a grid size of $\Delta\eta$ as 0.01. In order to study the effects of the coupling number K, magnetic field parameter M, suction/injection parameter $f_0$ and thermal stratification parameter $\varepsilon_1$ on the physical quantities of the flow.

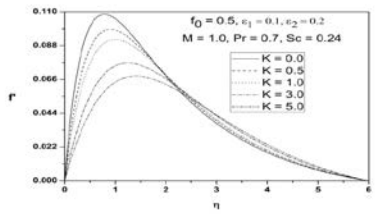


Fig.1(a)

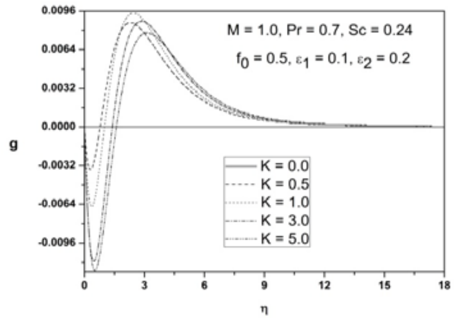


Fig.1(b)

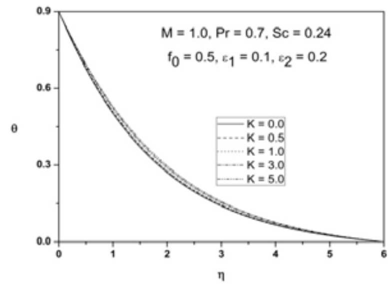


Fig.1(c)

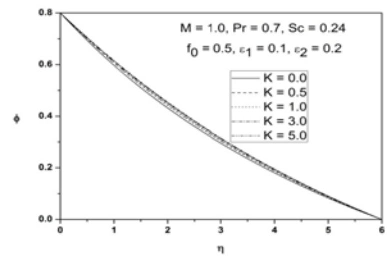


Fig.1(d)

Fig.1. (a) Velocity, (b) Micro rotation, (c) Temperature and (d) Concentration Profiles for various values of coupling parameter K

Fig. 1 depicts the variation of coupling number (K) on the profiles of velocity, micro rotation, temperature and concentration. The coupling number K characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, K signifies the coupling between the Newtonian and rotational viscosities[7]. As K increases, the effect of microstructure becomes significant, whereas with a small value of K the individuality of the substructure is much less pronounced. As $\kappa \to 0$ i.e. K → 0, the micropolarity is lost and the fluid behaves as non-polar fluid and thus the case of K → 0 corresponds to the case of viscous fluid. It is observed from Fig.1(a) that the velocity decreases with the increase of K. The maximum of velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall with an increase of K. The velocity in case of micro polar fluid is less than that in the viscous fluid case (K → 0 corresponds to viscous fluid). It is seen from Fig.1(b) that the micro rotation component decreases near the vertical plate and increases far away from the plate with increasing coupling number K. The microrotation tends to zero as K → 0 as is expected that in the limit $\kappa \to 0$, i.e. K → 0 the Eqs. (1) and (2) are uncoupled with Eq. (3) and they reduce to viscous fluid flow equations. It is noticed from Fig. 2(a) that the temperature increases with increasing values of coupling number.

The variation of the non-dimensional velocity, microrotation, temperature and concentration profiles with $\eta$ for different values of magnetic parameter is illustrated in Fig. 2. It is observed from Fig. 2(a) that velocity decreases as the magnetic parameter (M) increase

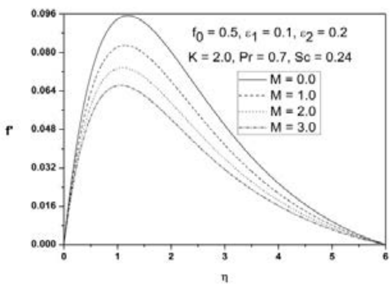


Fig.2(a)

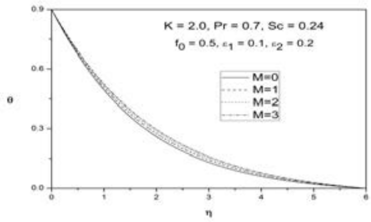


Fig.2(b)

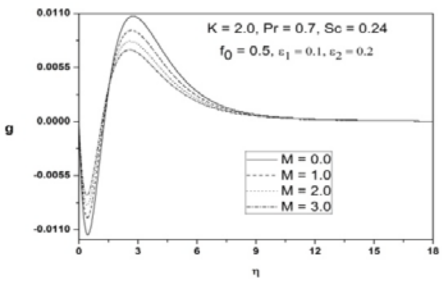


Fig.2(c)

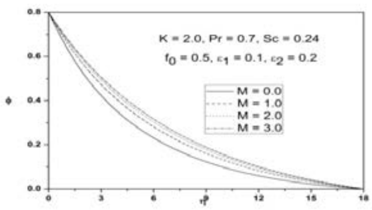


Fig.2(d)

Fig.2. (a) Velocity, (b) Micro rotation, (c) Temperature and (d) Concentration Profiles for various values of magnetic parameter M

from fig. 2(b), it is clear that the microrotation component increases near the plate and deceases far away from the plate for increasing values of m. it is noticed from fig. 2(c) that the non-dimensional fluid temperature increases with increasing values of magnetic parameter. it is clear from fig. 2(d) that the non-dimensional fluid concentration increases with increasing values of m. application of a uniform magnetic field normal to the flow direction produces a force which acts in the negative direction of flow. this force is called the lorentz force which tends to slow down the movement of the electrically conducting fluid in the vertical direction. this retardation effect is accompanied by an appreciable increase in the fluid temperature and concentration[8]. these behaviors are clearly depicted in fig. 2(c) and fig. 2(d).

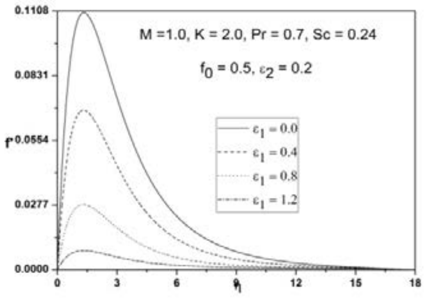


Fig. 3(a)

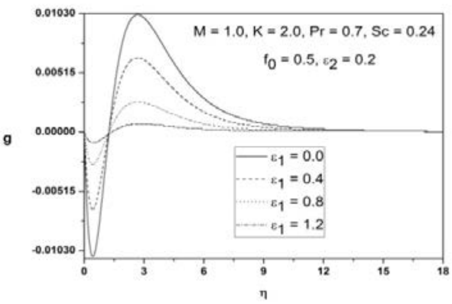


Fig.3(b)

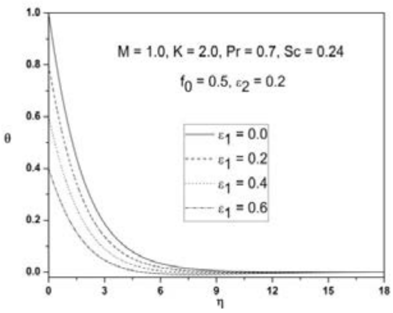


Fig.3(c)

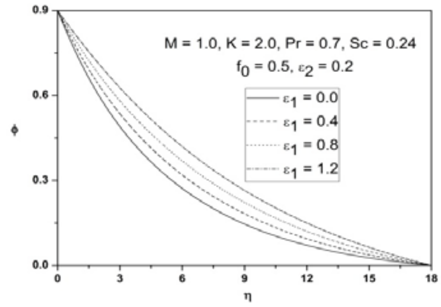


Fig.3.(d)

Fig.3. (a) Velocity, (b) Micro rotation, (c) Temperature and (d) Concentration Profiles for various values of thermal stratification parameter $\varepsilon_1$

The effect of thermal stratification parameter $\varepsilon_1$ on the non-dimensional velocity, microrotation, temperature and concentration is shown in Fig 3. It is observed from Fig. 3(a) that the velocity decreases with the increase of thermal stratification $\varepsilon_1$. This is because thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. From Fig. 3(b), we observe that the values of microrotation change sign from negative to positive at the critical point $\eta$= 0:8917 within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an increase in thermal stratification parameter. It is noticed from Fig. 3(c) that the non-dimensional temperature of the fluid decreases with the increase of thermal stratification parameter. When the thermal stratification effect is taken into consideration, the effective temperature difference between the plate and the ambient fluid will decrease; therefore, the thermal boundary layer is thickened and the temperature is reduced. Fig. 3(d) demonstrates that the concentration of the fluid increases with the increase of thermal stratification parameter. It can be noted that the effect of the stratification on temperature is the formation of a region with a temperature deficit (i.e., a negative dimensionless temperature).

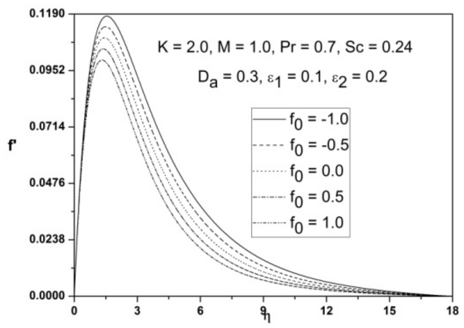


Fig.4(a)

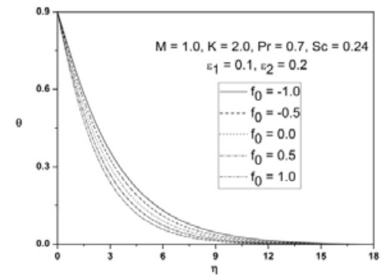


Fig.4(b)

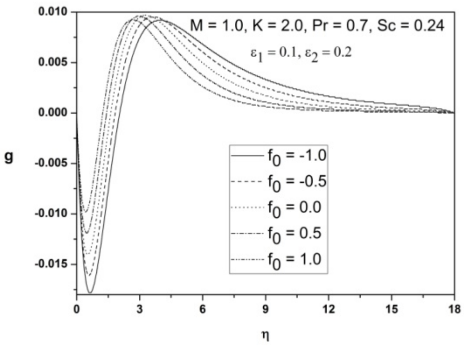


Fig.4(c)

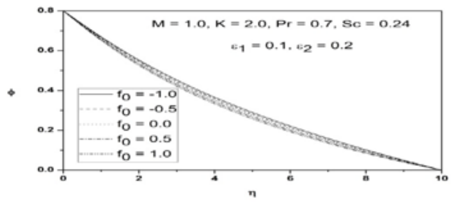


Fig.4(d)

Fig. 4(a) Velocity, (b) Micro rotation, (c) Temperature and (d) Concentration Profiles for various values of suction/injection.

Fig. 4 explains the case of porous vertical plate embedded in a doubly stratified micro polar saturated porous medium. In this case we considered the effect of suction/injection parameter $f_0$ on non-dimensional velocity, micro rotation, temperature and concentration profiles. From Fig. 4(a), as one may expect, that the fluid velocity is suppressed, i.e., the hydrodynamic boundary layer near the plate tends to become thinner, as suction velocity ($f_0 > 0$) is increasing. The novelty of this effect can be explained by the fact that the wall velocity gradient is decreased by sucking fluid particles through the porous wall, for this reason, the growth of fluid boundary layer is reduced. As a consequence, as suction velocity is increasing it is found to be restrict the velocity profile and the tendency is more pronounced towards to the plate. But it is interesting to notice that the opposite trend occurs in the case of injection (f0 < 0). The hydrodynamic boundary layer thickness is increased and broadens the velocity distribution as injection velocity increases. This is because of, the wall velocity gradient is increased when fluid injected in to the boundary layer. It is also clear that, for a blowing through the plate, the fluid behave qualitatively opposite to a suction velocity and the suction stabilizes the boundary layer thickness. From Fig. 4(b), it can be observed that the micro rotation component decreases near the vertical plate and increases far away from the plate with increasing $f_0$, showing a reverse rotation near the two boundaries. The reason is that the micro-rotation field in this region is dominated by a small number of particles which spin by collisions with the boundary. From injection to suction the microrotation is decreasing. From Fig. 4(c) it is clear that the temperature decreases as the suction parameter $f_0$ increases. Sucking the fluid particles through the porous wall brought the fluid closer to the wall and causes to reduce the growth of thermal boundary layer, consequently the temperature profiles are found to be suppressed. On the other hand as the velocity of injection is increasing the temperature profiles broaden and decrease the wall temperature gradient. Thus suction is better than blowing for cooling the surface much faster. It can be observed from Fig 4(d) that the concentration profiles suppressed and the tendency is more pronounced towards to the plate as suction velocity increases. i.e., sucking fluid particles from porous wall reduce the concentration of the fluid within the boundary layer. Whereas the same is increasing with the increase in injection velocity. Injecting the fluid into the concentration boundary layer through the porous wall increases the concentration.

## 8. CONCLUSION

- In this paper, a boundary layer analysis for free convection heat and mass transfer in an electrically conducting micro-polar fluid over a vertical plate with variable temperature and concentration wall conditions in the presence of a uniform magnetic field of magnitude $B_0$ and thermal stratification effects is performed.
- Using the similarity variables, the governing equations are transformed into a set of parabolic equations and numerical solution for these equations has been presented for different values of parameters.
- The higher values of the coupling number K (i.e., the effect of micro rotation becomes significant) result in lower velocity distribution but higher wall temperature; wall concentration distributions in the boundary layer compared to the Newtonian fluid case.
- The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the micro-polar fluid is lower compared to that of the Newtonian fluid.
- The present analysis has also shown that the flow field is appreciably influenced by the Magnetic parameter (M), Thermal stratification parameter parameter.

## REFERENCE


[1] Eringen AC. Theory of micro-polar fluids. J Math Mech 1966; 16: 1.18.

[2] Lukaszewicz G. Micro-polar Fluids: Theory and Applications. Basel, Switzerland: Birkhauser, 1999.

[3] Alam MS, Rahman MM, Sattar MA. MHD free convection heat and mass transfer past a moving vertical porous plate. Int Common Heat Mass 2005; 32: 1411{1419.

[4] Abdelkhalek MM. Heat and mass transfer in MHD free convection from a moving permeable vertical surface by a perturbation technique. Common Nonlinear Sci 2009; 14: 2091{2102.

[5] Ahmadi G. Self-similar solution of incompressible micro-polar boundary layer over a semi-infnite plate. Int J Eng Sci 1976; 14: 639{646.

[6] Takhar HS, Soundalgekar VM. Heat transfer on a semi-infinite plate of micropolar fluid. Rheol Acta 1980; 19: 525.526.

[7] Agarwal RS, Dhanapal C. Flow and heat transfer in a micropolar fluid past a plate with suction and heat sources. Int J Eng Sci 1988; 26: 1257{1266.

[8] Gorla RSR, Slaouti A, Takhar HS. Free convection in micropolar fluids over a uniformly heated vertical plate. Int J Numer Method H 1998; 8: 504{518.