

MODEL ORDER REDUCTION USING IMPROVED POLE CLUSTERING TECHNIQUE

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Abstract— Modeling physical systems are usually results in complex high-order dynamic models. It is necessary to reduce it to a lower order system. A mixed method is suggested for reducing order of the large scale interval systems. Kharitonov polynomial is employed before the order reduction is come into the approximation process. The denominator polynomial of the reduced order is obtained by the improved pole clustering technique while numerator polynomial of reduced order is determined through the pade approximation method. The reduced order model so obtained preserves the stability of the higher order system. The proposed method is validated by numerical examples from the literature.

Keywords—Improved Pole Clustering; Integral Square Error (ISE); Kharitonov theorem; Model Order Reduction; Pade Approximation

1. INTRODUCTION

The determination of the performance and stability of the system is the major concern of the control engineers. The majority of system coefficients are uncertain, and these coefficients are varying between the intervals. The systems whose coefficients are varied in a specified manner within an interval are called interval systems. Every coefficient will have the lower and upper limit. Modeling physical system usually results in complex high order dynamic models. The reduction of high order system (Large scale system) into low order is one of the important problems in control and system theory and is considered essential in analysis, synthesis and simulation of practical system. The low order model. The precise analysis of high order system is both tedious and costly.

Several applications involving signal processing, controlling chemical plants, nuclear reactor and process industries increase the importance of the reduced order model. Methods to reduce the order of systems are often utilized for alleviating computational difficulty, clarifying system analysis, and thus minimizing time and costs. Numerous methods have been suggested for the order reduction of continuous and discrete time systems [1]-[7]. Several methods for order reduction include Routh approximation, Moment matching technique, Pade approximation, Aggregation method, and Pole clustering techniques. The pade approximation technique have simple features such as computation simplicity and fitting of the time moments [7],[8]. A vital drawback of this method is that it sometimes leads to an unstable reduced order model. The methods have been suggested for model order reduction of interval system [8]-[15].

In this paper, a mixed method for order of higher order interval system in which the denominator of the model is determined by improved pole clustering technique and the corresponding numerator is obtained by pade approximation process. The suggested method is compared and evaluated.

2. KHARITONOV THEOREM

The Kharitonov theorem can be called as an extension on Routh stability criterion to interval polynomial. The Kharitonov theorem states that an interval polynomial family, which has an infinite number of members, is Hurwitz stable if and only if a finite small set of four polynomials known as Kharitonov Polynomials of the family are Hurwitz stable.

Consider a nth order polynomial of the form,

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0$$

with all real coefficients. The polynomial P(s) is said to be Hurwitz stable if all of its zeros belong to left half of the splane. Let the coefficients are bounded between upper and

lower limits as
$$\underline{a_i} < a_i < \overline{a_i}$$
 i=0,1,2...n-1, where $\underline{a_i}$

represents the lower limit and a_i represents the upper limit.

According to the Kharitonov theorem, the Hurwitz stability of just four polynomials selected from set 'N' guarantees stability of all polynomials belong to 'N' where N is the set of all polynomials whose coefficients belong to specified intervals.

The polynomial P(s) is split into two stable polynomial g(s) and h(s) where, g(s) is polynomial of even degree and h(s) is polynomial of odd degree. Defining two even polynomials,

$$K_{1}^{even,\min}(S) = g_{1}(s) = a_{0} + a_{2}s^{2} + a_{4}s^{4} + \dots$$
$$K_{1}^{even,\max}(S) = g_{2}(s) = a_{0} + a_{2}s^{2} + a_{4}s^{4} + \dots$$

Defining two odd polynomials,

$$K_{1}^{odd,\min}(S) = h_{1}(s) = a_{1}s + a_{3}s^{3} + a_{5}s^{5} + \dots$$
$$K_{1}^{odd,\max}(S) = h_{2}(s) = a_{1}s + a_{3}s^{3} + a_{5}s^{5} + \dots$$

IJRME - International Journal of Research in Mechanical Engineering Volume: 01 Issue: 02 2014 www.researchscript.com



The four Kharitonov polynomials would be

$$K_{11}(S) = g_1(s) + h_1(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \dots$$

$$K_{12}(S) = g_1(s) + h_2(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \dots$$

$$K_{21}(S) = g_2(s) + h_1(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \dots$$

$$K_{22}(S) = g_2(s) + h_2(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \dots$$

The main drawback on application of Kharitonov theorem is that it cannot be applied to polynomials with affine linear uncertainty structures. For such systems the Generalized Kharitonov theorem provide solution. The Generalized Kharitonovtheorem states that it is necessary to check the entire segment to prove the system stability.

3. PROBLEM FORMULATION

Let the transfer function of higher order interval system of the order 'n' is given by

$$G(s) = \frac{[a_{n-1}, a_{n-1}]s^{n-1} + [a_{n-2}, a_{n-2}]s^{n-2} + \dots + [a_1, a_1]s + [a_0, a_0]}{[b_n, \bar{b_n}]s^n + [b_{n-1}, \bar{b_{n-1}}]s^{n-1} + [b_{n-2}, \bar{b_{n-2}}]s^{n-2} + \dots + [b_1, \bar{b_1}]s + [b_0, \bar{b_0}]} = \frac{N(s)}{D(s)}$$

where, $\begin{bmatrix} a \\ -i \end{bmatrix}$, $\bar{a}_i \end{bmatrix}$ for i=0 to n-1 and $\begin{bmatrix} b \\ -i \end{bmatrix}$, $\bar{b}_i \end{bmatrix}$ for i=0 to n are

the interval coefficient of higher order numerator and denominator polynomial respectively.

Let the corresponding reduced order model is obtained as

$$G_{r}(s) = \frac{\begin{bmatrix} d_{0}, \bar{d}_{0} \end{bmatrix} + \begin{bmatrix} d_{1}, \bar{d}_{1} \end{bmatrix} s + \dots + \begin{bmatrix} d_{-k-1}, \bar{d}_{k-1} \end{bmatrix} s^{k-1}}{\begin{bmatrix} e_{0}, \bar{e}_{0} \end{bmatrix} + \begin{bmatrix} e_{1}, \bar{e}_{1} \end{bmatrix} s + \dots + \begin{bmatrix} e_{-k-1}, \bar{e}_{k-1} \end{bmatrix} s^{k-1} + \begin{bmatrix} e_{-k}, \bar{e}_{k} \end{bmatrix} s^{k}} = \frac{N_{r}(s)}{D_{r}(s)}$$

where, $[d_i, d_i]$ for i=0 to n-1 and $[e_i, e_i]$ for i=0 to n

are the interval coefficient of lower order numerator and denominator polynomial respectively. The objective is to obtain a reduced second order model for an interval system that retains and reflects the important characteristics of the original system as closely as possible.

4. PROPOSED METHOD

The proposed Model Order Reduction methods consists of three steps.

Step 1: Determination of the reduced order denominator polynomial using improved pole clustering technique.

Calculate the 'n' number of poles from the given higher order system denominator polynomial.

The number of cluster centers to be calculated is equal to the order of the reduced system. The poles are distributed into the cluster center for the calculation such that none of the repeated poles present in the same cluster center. The minimum number of poles distributed per each cluster center is at least one. There is no limitation for the maximum number poles per cluster center. Let k number of poles be available in a cluster center: $p_1, p_2, p_3, \dots, p_k$. The poles are arranged in a manner such that $|p_1| > |p_2|, \dots, |p_k|$. The cluster center for the reduced order model can be obtained by using the following **Research script | LJRME Volume: 01 Issue: 02 2014 (C)**

procedure. The procedure described in step 1 is similar to the case of the method proposed by [13] but the pole cluster calculated in the proposed scenario is based on the dominant pole in that particular cluster center.

Let k number of poles isavailable $|p_1| < |p_2| ... < |p_k|$. Find the pole cluster as,

$$C_{L} = [(-1/|p_{1}| + \sum_{i=2}^{k} -1/|p_{i} - p_{1}|) \div k]^{-1},$$

The improved cluster centre form

$$C_L = [(-1/|p_1| + -1/|C_L|) \div 2]^{-1}.$$

While calculating the cluster center values, we have the following three cases as in [13].

Case 1: When all the denominator poles are real, the corresponding reduced order denominator polynomial can be obtained as,

$$D_k(s) = (s - p_{c1})(s - p_{c2})....(s - p_{ck})$$

Case 2: If one pair of cluster center is complex conjugate and (k-2) cluster center are real.

$$D_{k}(s) = \left(s - \left(p_{1}^{R} + jp_{1}^{I}\right)\right)\left(s - p_{1}^{R} - jp_{1}^{I}\left(s - p_{ci}\right)\right)$$

Case 3: Whenall the denominatorpoles are complexconjugates, the corresponding reduced order denominator polynomial can be obtained as,

$$D_{k}(s) = \left(s - \left(p_{1}^{R} + jp_{1}^{I}\right)\right)\left(s - \left(p_{1}^{R} - jp_{1}^{I}\right)\right)$$

Step 2: Determination of the reduced order numerator polynomial using first order pade approximation.

On equating and cross multiplying the transfer functions G(s) and $G_r(s)$, (r+2) number of equations are obtained. On solving those equations remaining numerator polynomial unknown parameters can be calculated.

$$\frac{a_0 + a_1s + a_2s^2 + \dots + a_{n-1}s^{n-1}}{b_0 + b_1s + b_2s^2 + \dots + b_{n-1}s^{n-1} + b_ns^n} = \frac{d_0 + d_1s + d_2s^2 + \dots + d_{r-1}s^{r-1}}{e_0 + e_1s + e_2s^2 + \dots + e_{r-1}s^{r-1} + e_rs^r}$$

$$\frac{d_0 + d_1s + d_2s^2 + \dots + d_{r-1}s^{r-1}}{e_0 + e_1s + e_2s^2 + \dots + e_{r-1}s^{r-1} + e_rs^r}$$

Step 3: Adjustment of reduced order model coefficients using GA

The coefficients of reduced order model are adjusted or tuned by using the Genetic Algorithm as detailed in [13].

5. NUMERICAL ILLUSTRATION

The 3rd order interval system stated in [12] is considered as

$$G(s) = \frac{[2,3]s^2 + [17.5,18.5]s + [15,16]}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5,21.5]}$$

The given interval system transfer function is in the form of

$$G(s) = \frac{[a_2, a_2]s^2 + [a_1, a_1]s + [a_0, a_0]}{[b_3, b_3]s^3 + [b_2, b_2]s^2 + [b_1, b_1]s + [b_0, b_0]} = \frac{N(s)}{D(s)}$$

The reduced order interval system is to be derived in the form of

$$G_2(s) = \frac{[d_1, d_1]s + [d_0, d_0]}{[e_2, e_2]s^2 + [e_1, e_1]s + [e_0, e_0]}$$

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From the given interval system, the upper and lower limit values are noted and are used to obtain the Kharitonov Polynomials.

$$a_0 = 15$$
 $a_0 = 16$ $a_1 = 17.5$ $a_1 = 18.5$ $a_2 = 2$ $a_2 = 3$ $b_0 = 20.5$ $b_0 = 21.5$ $b_1 = 35$ $b_1 = 36$ $b_2 = 17$ $b_2 = 18$ $b_3 = 2$ $b_3 = 3$

Instead of applying the model order reduction procedure to single interval system as a whole the system can be split up to 4 Kharitonov polynomials of the denominator and 4 Kharitonov polynomials of the numerator. After arriving at the reduced denominator and each denominator has 4 possible combinations of the numerator and for the 4 denominator functions totally 16 possible system models are obtained. The numerator of the four Kharitonov polynomials are,

$$N_{1}(s) = K_{11}(s) = g_{1}(s) + h_{1}(s) = a_{0} + a_{1}s + a_{2}s^{2} = 15 + 17.5s + 3s^{2}$$

$$N_{2}(s) = K_{12}(s) = g_{1}(s) + h_{2}(s) = a_{0} + a_{1}s + a_{2}s^{2} = 15 + 18.5s + 3s^{2}$$

$$N_{3}(s) = K_{21}(S) = g_{2}(s) + h_{1}(s) = a_{0} + a_{1}s + a_{2}s^{2} = 16 + 17.5s + 2s^{2}$$

$$N_{4}(s) = K_{22}(S) = g_{2}(s) + h_{2}(s) = a_{0} + a_{1}s + a_{2}s^{2} = 16 + 18.5s + 2s^{2}$$

And the corresponding denominator Kharitonov Polynomial is,

$$D_{1}(s) = K_{11}(s) = g_{1}(s) + h_{1}(s) = b_{0} + b_{1}s + b_{2}s^{2} + b_{3}s^{3} = 20.5 + 35s + 18s^{2} + 3s^{3}$$

$$D_{2}(s) = K_{12}(s) = g_{1}(s) + h_{2}(s) = b_{0} + b_{1}s + b_{2}s^{2} + b_{3}s^{3} = 20.5 + 36s + 18s^{2} + 2s^{3}$$

$$D_{3}(s) = K_{21}(s) = g_{2}(s) + h_{1}(s) = b_{0} + b_{1}s + b_{2}s^{2} + b_{3}s^{3} = 21.5 + 35s + 17s^{2} + 3s^{3}$$

$$D_{4}(s) = K_{22}(s) = g_{2}(s) + h_{2}(s) = b_{0} + b_{1}s + b_{2}s^{2} + b_{3}s^{3} = 21.5 + 36s + 17s^{2} + 2s^{3}$$

From the Kharitonov polynomials available for numerator And denominator of higher order interval system, the following four interval system transfer function may be applied.

$$G_{1}(s) = \frac{N_{1}(s)}{D_{1}(s)} = \frac{a_{0} + a_{1}s + a_{2}s^{2}}{b_{0} + b_{1}s + b_{2}s^{2} + b_{3}s^{3}} = \frac{15 + 17.5s + 3s^{2}}{20.5 + 35s + 18s^{2} + 3s^{3}}$$
$$G_{2}(s) = \frac{N_{2}(s)}{D_{2}(s)} = \frac{a_{0} + a_{1}s + a_{2}s^{2}}{b_{0} + b_{1}s + b_{2}s^{2} + b_{3}s^{3}} = \frac{15 + 18.5s + 3s^{2}}{20.5 + 36s + 18s^{2} + 2s^{3}}$$

$$G_{3}(s) = \frac{N_{3}(s)}{D_{3}(s)} = \frac{\overline{a_{0}^{+} + a_{1}^{-} s + a_{2}^{-} s^{2}}}{\overline{b_{0}^{+} + b_{1}^{-} s + b_{2}^{-} s^{2} + \overline{b_{3}^{-} s^{3}}}} = \frac{16 + 17.5s + 2s^{2}}{21.5 + 35s + 17s^{2} + 3s^{3}}$$
$$G_{4}(s) = \frac{N_{4}(s)}{D_{4}(s)} = \frac{\overline{a_{0}^{+} + a_{1}^{-} s + a_{2}^{-} s^{2}}}{\overline{b_{0}^{+} + b_{1}^{-} s + b_{2}^{-} s^{2} + b_{3}^{-} s^{3}}} = \frac{16 + 18.5s + 2s^{2}}{21.5 + 36s + 17s^{2} + 2s^{3}}$$

The proposed model order reduction method is applied for the system interval functions. The following reduced order model are obtained.

$$G_{r1}(s) = \frac{d_{1}s + d_{0}}{\bar{e_{2}}s^{2} + e_{1}s + e_{0}} = \frac{1.529s + 2.0473}{s^{2} + 3.6008s + 2.798}$$

$$G_{r2}(s) = \frac{\bar{d_{1}}s + d_{0}}{\bar{e_{2}}s^{2} + \bar{e_{1}}s + e_{0}} = \frac{1.337s + 1.4137}{s^{2} + 2.8533s + 1.9319}$$

$$G_{r3}(s) = \frac{\bar{d_{1}}s + \bar{d_{0}}}{\bar{e_{2}}s^{2} + e_{1}s + \bar{e_{0}}} = \frac{1.6074s + 2.224}{s^{2} + 3.7549s + 2.9883}$$

$$G_{r4}(s) = \frac{\bar{d_{1}}s + \bar{d_{0}}}{e_{2}s^{2} + \bar{e_{1}}s + \bar{e_{0}}} = \frac{1.4806s + 1.633}{s^{2} + 3.1264s + 2.1994}$$

From the reduced order system transfer function available in the above equation, the following conditions are obtained.

$$d_{0} = \min\{2.0473, 1.4137, 2.224, 1.633\} = 1.4137$$

$$d_{1} = \min\{1.529, 1.337, 1.6074, 1.4806\} = 1.337$$

$$e_{0} = \min\{2.798, 1.9319, 2.9883, 2.1994\} = 1.9319$$

$$e_{1} = \min\{3.6008, 2.8533, 3.7549, 3.1264\} = 2.8533$$

$$e_{2} = \min\{1, 1, 1, 1\} = 1$$

$$\bar{d}_{0} = \max\{2.0473, 1.4137, 2.224, 1.633\} = 2.224$$

$$\bar{d}_{1} = \max\{1.529, 1.337, 1.6074, 1.4806\} = 1.6074$$

$$\bar{e}_{0} = \max\{2.798, 1.9319, 2.9883, 2.1994\} = 2.9883$$

$$\bar{e}_{1} = \max\{3.6008, 2.8533, 3.7549, 3.1264\} = 3.7549$$

$$e_2 = \max\{1, 1, 1, 1\} = 1$$

The reduced order interval system transfer function can be obtained as,

$$G_{n}(s) = \frac{[d_{1}, \bar{d}_{1}]s + [d_{0}, \bar{d}_{0}]}{[e_{2}, \bar{e}_{2}]s^{2} + [e_{1}, \bar{e}_{1}]s + [e_{0}, \bar{e}_{0}]} = \frac{[1.337, 1.6074]s + [1.4137, 2.224]}{[1, 1]s^{2} + [2.8533, 3.7549]s + [1.9319, 2.9883]}$$
By

applying the GA in tuning coefficients of reduced order interval system coefficients, the following reduced order interval system is obtained.

$$G_r(s) = \frac{[1.337, 1.6074]s + [1.4386, 2.3945]}{[1,1]s^2 + [2.8662, 3.7553]s + [1.9656, 3.2151]}$$

The variation in the error rate over the iterations for lower and upper limit interval systems is shown in Figure 1 and Figure 2 respectively. The step response of higher and lower order interval system through improved pole clustering technique are shown in Figure 3.



Fig.1. Change in error rate of lower limit interval system using GA tuning



Fig.2. Change in error rate of upper limit interval system using GA tuning The performance comparison of the proposed order reduction technique with pole clustering techniques proposed by [15] is listed in Table 1. The comparison of pole clustering and improved pole clustering is made by computing the error index ISE between the transient parts of the original and reduced order model and is calculated to measure the goodness/quality of the reduced order model (i.e. the smaller the ISE, the closer is $G_r(s)$ to G(s)).

$$ISE = \int_{0}^{t_{\infty}} [y(t) - y_r(t)]^2 dt$$

where y(t) and $y_r(t)$ are the unit step responses of original and reduced order systems for a second- order reduced respectively. It can be seen that the steady state responses of all the reduced order models are exactly matching with that of the original model.



Fig.3. Step response of higher and reduced order interval system via improved pole clustering technique.

TABLE.1. COMPARISON OF ERROR INDEX VALUES WITH EXISTING POLE CLUSTERING METHOD FOR EXAMPLE 1

Reduction Method	ISE	
	Lower limit	Upper limit
Pole Clustering method [15]	0.0004248	0.000146
Improved pole clustering method	0.00007085	0.0002053

6. STABILITY ANALYSIS

Routh-Hurwitz criterion is easily used to check the stability of a simple polynomial which becomes difficult to apply to families of polynomials because it gives nonlinear equations with the unknown parameters. The Kharitonov theorem states that the stability of a polynomial can be determined by testing just the four Kharitonov polynomials which are obtained by using upper and lower bounds of the unknown parameters. Applying the Routh-Hurwitz criterion to the selected Kharitonov polynomials of given interval system. If the elements in the first column of the Routh array are positive then the interval system is stable, otherwise not.

The 3rd order interval system stated in [12] is considered as $[2,3]s^2 + [17.5,18.5]s + [15,16]$

$$G(s) = \frac{1}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5,21.5]}$$

The corresponding characteristic polynomial P(s) is obtained as,

$$1 + \frac{[2,3]s^2 + [17.5,18.5]s + [15,16]}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5,21.5]} = 0$$

$$P(s) = [2,3]s^3 + [19,21]s^2 + [52.5,54.5]s + [35.5,37.5]$$

The polynomial P(s) is Hurwitz stable, if and only if the following polynomials are Hurwitz stable according to the Kharitonov theorem. The Kharitonov polynomial of polynomial is obtained.

$$K_{11}(S) = g_1(s) + h_1(s) = 3s^3 + 21s^2 + 52.5s + 35.5$$

$$K_{12}(S) = g_1(s) + h_2(s) = 2s^3 + 21s^2 + 54.5s + 35.5$$

$$K_{21}(S) = g_2(s) + h_1(s) = 3s^3 + 19s^2 + 52.5s + 37.5$$

$$K_{22}(S) = g_2(s) + h_2(s) = 2s^3 + 19s^2 + 54.5s + 37.5$$

These polynomials are called Kharitonov polynomials. The midpoint of the interval system is given by,

$$K(S) = 2.5s^3 + 20s^2 + 53.5s + 36.5s$$

The stability of the above nominal system is determined by applying the Routh-Hurwitz criterion and is detailed in Table 2. Since there is no sign change in the first column of Routh array, there are no roots in the right half of the splane. Hence, the system is stable. For each of the four polynomials, Routh array can be obtained and there is no sign change in the elements of first column of the array. This indicates the interval system is stable. The same procedure may be repeated to check the stability of reduced order model.



TABLE 2 ROUTH-HURWITZ TABLE FOR NOMINAL KHARITONOV POLYNOMIAL

S ³	2.5	53.5
S^2	20	36.5
S^1	48.9375	0
S^0	36.5	

Consider the reduced order system transfer function obtained as,

$$G_{r2}(s) = \frac{d_1 s + d_0}{e_2 s^2 + e_1 s + e_0} = \frac{1.337 s + 1.4137}{s^2 + 2.8533 s + 1.9319}$$

The corresponding characteristic equation is given by,

 $s^{2} + 2.8533s + 1.9319 = 0$ TABLE 3 ROUTH-HURWITZ TABLE FOR REDUCED ORDER

SYSTEM

S^2	1	1.9319
S^1	2.8533	0
S ⁰	1.9319	

The Routh-Hurwitz table for the above characteristic equation is given in Table 3. In this table, there is no sign change in the elements of first column of the array. This indicates the reduced order system is stable. In a similar, the stability can be analyzed for remaining three possible groups of Kharitonov polynomial of transfer functions. Thus the reduced order models of interval systems obtained through the proposed mixed methods are stable.

7. CONCLUSION

In this paper, an improved pole clustering technique for an interval system is proposed to obtain the reduced order model. The closeness between the original and reduced order model is analyzed with help of ISE values. The proposed scenario gives a better result as compared with pole clustering technique. The stability of the original and reduced order interval systems were checked by Routh-Hurwitz criterion. This method assures the stability in the reduced order model, provided the given higher order model. The proposed technique may be extended for analysis and design a compensator for Large scale interval system.

8. FUTURE WORK

The Proposed Technique maybe extended for analysis and design of a compensator for large scale systems.

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